Equations with Fractions

We have experience solving equations that have fractions, especially those equations where the variable is multiplied by a fraction. Below is an example, solving $\frac{2}{3}x = 12$.

$$\frac{2}{3}x = 12$$
$$\cdot\tfrac{3}{2} \quad \cdot\tfrac{3}{2} \qquad \longrightarrow \quad \text{Multiply both sides of the equation by } \tfrac{3}{2}.$$
$$x = 18$$

☺

However, there are plenty of equations with addition and subtraction that also have fractions. In these cases, just solve the equation like you solve all other equations with addition and subtraction: by using the opposite operation. Below is an example, solving $x + \frac{2}{3} = 12$.

$$x + \frac{2}{3} = 12$$
$$-\tfrac{2}{3} \quad -\tfrac{2}{3} \qquad \longrightarrow \quad \text{Subtract } \tfrac{2}{3} \text{ from each side of the equation.}$$
$$x = \frac{34}{3}$$

☺

Notice that we did not use the reciprocal in the equation above. Instead, we just solved the equation using subtraction. Below is another example, solving $-\frac{2}{5} + x = -\frac{2}{3}$.

$$-\frac{2}{5} + x = -\frac{2}{3}$$
$$+\tfrac{2}{5} \qquad +\tfrac{2}{5} \qquad \longrightarrow \quad \text{Add } \tfrac{2}{5} \text{ to each side of the equation.}$$
$$x = -\frac{4}{15}$$

☺

Again, notice that this equation does not require us to use the reciprocal. Also notice that we added to each side of the equation to remove the negative number, even though the equation already contains addition.

Below are a mix of equations that contain fractions. Let's solve them together:

- $\frac{1}{2}x = 4$
- $x + \frac{1}{2} = 4$
- $x - \frac{1}{2} = 4$

- $\frac{3}{5} + x = 9$
- $-\frac{3}{5} + x = 9$
- $\frac{3}{5}x = 9$

Now it's your turn to practice.

1. Use addition and subtraction to solve each equation.

(a) $x + 5 = -100$ **(b)** $x - 4 = -31$ **(c)** $x + \frac{7}{3} = 22$

(d) $-3 + x = 119$ **(e)** $-\frac{1}{3} + x = 2$ **(f)** $\frac{4}{5} + x = \frac{1}{2}$

2. Use the reciprocal to solve each equation.

(a) $\frac{2}{7}x = 2$ **(b)** $\frac{-8}{9}x = -16$ **(c)** $\frac{3}{-4}x = 13$

3. Solve each equation.

(a) $-9 + x = 36$ **(b)** $\frac{2}{9}x = -10$ **(c)** $8 + x = \frac{3}{4}$

(d) $5 = \frac{4}{3} + x$ **(e)** $\frac{3}{8}x = 90$ **(f)** $\frac{4}{9} = -\frac{1}{2} + x$