Using Substitution After Solving An Equation For A Variable, Again

Today we again work with exercises where neither equation is solved for a variable. This time, solving an equation for a variable won't be quite as pretty as before, but the process remains the same. Then we use that information to perform substitution as before.

Here's an example highlighting this process

Example: Use substitution to solve the system $\begin{cases} 2x+3y=24 \\ -5x+2y=-41 \end{cases}$.

Neither of these equations is solved for a variable, meaning neither equation looks like $y=$ or $x=$. So we need to choose one of the variables to solve for. This time, we cannot avoid the use of fractions, so it does not really matter which variable we solve for. Let's solve for y in the second equation:

$$-5x+2y=-41 \longrightarrow \text{The second equation.}$$
$$2y=5x-41 \longrightarrow \text{Adding } 5x \text{ to each side of the equation.}$$
$$y=\tfrac{5}{2}x-\tfrac{41}{2} \longrightarrow \text{Dividing each side of the equation by 2.}$$

Now we can use this slightly ugly information for y and substitute it into the first equation:

$$2x+3y=24 \longrightarrow \text{The first equation.}$$
$$2x+3\left(\tfrac{5}{2}x-\tfrac{41}{2}\right)=24 \longrightarrow \text{Substituting the information from the second equation.}$$
$$2x+\tfrac{15}{2}x-\tfrac{123}{2}=24 \longrightarrow \text{Distributing the 3 to remove the parentheses.}$$
$$4x+15x-123=48 \longrightarrow \text{Multiplying both sides of the equation by 2 to remove the}$$
$$19x-123=48 \longrightarrow \text{Combining like terms.}$$
$$19x=171 \longrightarrow \text{Adding 123 to each side of the equation.}$$
$$x=9 \longrightarrow \text{Dividing each side of the equation by 19.}$$

So now we know that the equations are equal when $x = 9$. However, we do not yet know the y value of the solution. To figure this out, we need to take our x value and substitute it into one of the original equations. We can substitute this $x = 9$ into either of the original equations, but we want to make sure we use an equation that has both xs and ys in it. In this case, let's just use the first equation in the system:

$$\begin{aligned} 2x + 3y &= 24 & \longrightarrow & \quad \text{The first equation.} \\ 2(9) + 3y &= 24 & \longrightarrow & \quad \text{Substituting } x = 9 \text{ into the first equation.} \\ 18 + 3y &= 24 & \longrightarrow & \quad \text{Multiplying to simplify the equation.} \\ y &= 2 & \longrightarrow & \quad \text{Solving for } y. \end{aligned}$$

So we know the solution to the system of equations is $(9, 2)$.

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Let's try some exercises together. Use substitution to solve each system of equations. Be sure to write your answers as an ordered pair (i.e., (x, y)).

- $\begin{cases} 10x + 4y = 5 \\ 20x + 100y = -13 \end{cases}$
- $\begin{cases} 15x - 5y = 79 \\ 15x - 15y = 87 \end{cases}$

Now it's your turn. Use substitution to solve each system of equations. Be sure to write your answers as an ordered pair (i.e., (x, y)).

1. $$\begin{cases} 28x + 21y = 29 \\ -70x + 42y = -5 \end{cases}$$

2. $$\begin{cases} -9x + 45y = 4 \\ -45x - 9y = 20 \end{cases}$$